A MODEL OF INVESTMENT UNDER UNCERTAINTY: MODERN IRRIGATION TECHNOLOGY AND EMERGING MARKETS IN WATER

JANIS M. CAREY AND DAVID ZILBERMAN

This article develops a stochastic dynamic model of irrigation technology adoption. It predicts that farms will not invest in modern technologies unless the expected present value of investment exceeds the cost by a potentially large hurdle rate. The article also demonstrates that, contrary to common belief, water markets can delay adoption. The introduction of a market should induce farms with abundant (scarce) water supplies to adopt earlier (later) than they would otherwise. This article was motivated by evidence that, contrary to NPV predictions, farms wait until random events such as drought drive returns significantly above costs before investing in modern irrigation technologies.

Key words: dynamic optimization, option value, technology adoption, water, water markets.

This article develops a dynamic technology adoption model with input supply and price uncertainty. The model examines the effect of a water market on a farm’s decision to adopt modern water-conserving irrigation technology. Due to the uncertainty of future water supplies and prices and the quasi-reversible nature of an investment in modern technology, the option to delay investment provided by a water market can be valuable. By waiting to invest, a farm can observe whether water prices increase or decrease before committing to a sunk investment cost.

There is an extensive literature on irrigation technology adoption; however it does not address the effects of uncertainty, irreversibility and the option to wait on a farm’s investment strategy (Caswell, Lichtenberg and Zilberman; Zilberman and Dinar). The traditional net present value (NPV) models of investment predict that a farm will invest when the expected present value of investment equals the cost of investment. A key result of this real options model is that a farm will not invest in modern technology until the expected present value of investment exceeds the cost by a potentially large hurdle rate. This article will demonstrate that the option value investment rule is more consistent with observed behavior than the NPV rule.

The second major result of this article is that the introduction of a water market can decrease technology adoption incentives for some farms. This result contradicts the common perception that water markets will always increase adoption rates (Dinar and Letey). If a farm has abundant water supplies, then the introduction of a water market should indeed increase its incentive to invest in modern technology. However, if a farm has scarce supplies, the introduction of a water market may cause it to postpone irrigation technology investments because it has the option to purchase water in a market.

The article employs option value theory developed in the finance literature and popularized by Dixit and Pindyck. Increasingly, authors have been applying option value theory to problems of so-called “real investment” (Abel and Eberly; Davis; Howitt; Hubbard; McDonald and Siegel). Applications of the option value approach include residential energy conservation investments (Hassett and Metcalf), electric utility compliance with SO2 emissions regulations (Herbelot), investments in free-stall dairy housing (Thurow et al.), and mineral asset pricing (Davis).

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The article proceeds as follows. The next section provides background on irrigation technology adoption and motivation for this article. Second, the model of irrigation technology adoption is presented. Third, the sensitivity of the farm’s investment strategy to changes in parameter values is analyzed, and policy implications are discussed. Fourth, a simulation is used to examine the timing of adoption predicted by the model. Fifth, the effects of barriers to trade are examined, and a farm’s incentives to adopt the modern technology with and without access to a water market are compared. The final section summarizes the key results of the article.

Background on Irrigation Technology Adoption

This research was motivated by the belief that traditional NPV models of investment do not accurately predict investment behavior. Many studies have provided evidence that modern irrigation technologies such as drip or sprinkler can yield higher expected profits than traditional technologies (University of California Committee of Consultants; McKenry). However, interviews with farm advisors indicate that they frequently observe “under-investment” in modern irrigation technologies. This conclusion that investment rates are suboptimal is based on the use of traditional NPV models of investment, which ignore issues of uncertainty and irreversibility. Using NPV, observed investment rates could only be considered optimal if farms were using extremely high discount rates.

Contrary to the predictions of NPV models, it appears that farms wait until the return on an investment is significantly greater than the cost before adopting modern irrigation technologies. Zilberman et al. found that adoption of drip irrigation increased dramatically during drought periods in California. From 1982 to 1986, adoption rates were slow even though modern technologies appeared to be cost effective in many areas. The five-year drought from 1987 to 1991 drove returns sufficiently above investment costs and triggered widespread adoption. The California drought intensified the adoption of modern technologies on crops that used them before, and led to their adoption on crops normally grown with traditional irrigation methods. Adoption of drip technology on fruit and vegetable crops increased by more than 40% and the range of crops irrigated with sprinklers also increased significantly.

These results are consistent with historical patterns of modern irrigation technology adoption in California. Drip irrigation was introduced in California in 1969, but adoption rates were very low (only about 40,000 acres) until 1976. The breakthrough for drip irrigation occurred during the drought of 1977–1979. The acreage of drip irrigation increased to 100,000 acres in 1977 and to 250,000 acres in 1979. The increase in drip acreage then tapered off from 1980 until the drought of 1988–1991 induced another flurry of investment (Caswell). Casterline found that nationwide changes in acreage of modern irrigation technologies were not gradual, but occurred mostly during brief periods associated with extreme events such as the California droughts or the high-energy prices of the 1970s that triggered investment in low-pressure, center-pivot irrigation in the Midwest.

The tendency to switch technologies in response to extreme events is not limited to irrigation. de Janvry, LeVeen, and Runsten found that, while the tomato harvester was available for quite some time, its large-scale adoption occurred in response to the termination of the Bracero Program when farmers faced higher labor costs and increased labor supply uncertainty. The widespread adoption of cotton harvesters was also associated with drastic changes in labor conditions.

The Model

The farm is assumed to maximize an instantaneous profit function by choosing the amount of water to apply to its crop. The farm initially produces with the traditional irrigation technology, but it has the option to invest in more efficient modern technology. If it switches technologies, the farm must pay an irreversible investment cost, which includes the cost of designing the system and investing in the new infrastructure (e.g., pipes, filters, and drainage equipment) and the cost associated with training workers to use the new irrigation system. Once the farm switches technologies, it is assumed to use the modern technology forever.1

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1 Future extensions of the model could incorporate the ability to switch back to the traditional technology or to an alternative
In each period $t$, the farm receives a stochastic supply of water, which can be adjusted by buying or selling water in a spot market. The market price of water is stochastic, and the market is assumed to be competitive with no transaction costs. Later, the article considers the implications of relaxing this assumption of frictionless trading.

The model captures the ability of a farm with access to a water spot market to adjust its water inputs in the short term in response to changes in the state of nature, while recognizing that the farm cannot easily change other production variables such as its irrigation technology. Thus, technology choice is a long-term decision, while the farm’s water market participation is a short-term decision.

A basic option-value investment problem, as described in the classic model developed by McDonald and Siegel and typical of most of the models in Dixit and Pindyck, does not specify a production function that would allow a farm to make short-term input adjustments. The McDonald and Siegel model focuses on output uncertainty, while this article analyzes input uncertainty—the farm’s initial water allocation and the price of water are stochastic. In addition, this article analyzes the decision of a farm to switch production technologies, so the farm generates a positive income flow both before and after the investment. Hassett and Metcalf provide another example of a technology-switching model. However, their model uses a constrained minimization framework.

Ideally, one might introduce further elements of realism. For example, the model might address the impact of output price variability on irrigation adoption decisions. However, in the context of the irrigation technology adoption decision, especially for farms in the semi-arid Western US, water price variability is more important than output price variability. This is especially true given the existence of farm price support programs that reduce output price variability. For example, the price of cotton has varied between 60 cents and one dollar per pound during the last 20 years, while the price of water can vary from 10 to 150 dollars per acre-foot in a single growing season (Slavin). Because of the complexity that already exists in the dynamic investment problem, the model assumes that output price is fixed.

### The Production Problem

Following the empirical findings of Letey and Berck and Helfand, we assume a von Liebig production function relating output to water input. Let $y_i$ be output per acre and $a_i$ be the applied water, in acre-feet per acre, with technology $i$. Then

$$ y_i = \begin{cases} h_i a_i & \text{for } a_i < a_i^* \\ y_i^* & \text{for } a_i \geq a_i^* \end{cases} i = 1, 2 $$

where $i = 1$ corresponds to the traditional technology and $i = 2$ corresponds to the modern technology. $h_i$ is the irrigation efficiency of technology $i$ in units per acre-foot. At lower water application levels, the yield per acre-foot of applied water is greater with the modern technology and thus $h_2 > h_1$. It is debated whether there is a difference in maximum output with the traditional and modern technologies. Following Letey and Berck and Helfand, we assume that $y_1^* = y_2^* = y^*$, and thus we do not have a yield-increasing effect associated with adoption if both technologies operate at full potential.

The farm’s profit flow is the outcome of an instantaneous optimization problem in which the farm chooses its water input holding the technology constant. The profit flow at $t$ is

$$ \pi_t(p, s) = \max_{a_i} \left( p^t h_i a_i - p(a_i - s) - w_i a_i - k_i \right), \quad i = 1, 2 $$

where $p^t$ is the output price in dollars per unit and $p$ is the market price of water. $s$ is the farm’s initial allocation of water, and $(a_i - s)$ is the amount of water the farm buys (sells) in the market when using technology $i$. Both $p$ and $s$ are assumed to be stochastic. Each technology requires a fixed cost per acre and marginal cost per acre-foot of water applied, denoted by $k_i$ and $w_i$, respectively. By assumption, $w_2 > w_1$ (reflecting higher pressurization costs) and $k_2 > k_1$. Given the von Liebig production function, if the farm produces a positive output, it will choose $a_i = a_i^*$.

To assess whether production is profitable, we evaluate a marginal and a total-profit condition. These conditions define the upper bound on $p$, above which the farm will shut down. The marginal condition is satisfied if

$$ \frac{\partial \pi_t}{\partial a_i} = p^t h_i - p - w_i \geq 0 \quad \text{for } a_i \leq a_i^* $$
implying an upper bound of \( \hat{p}_t = p^* h_t - w_t \) on the price of water. The condition states that the marginal cost of buying water must be less than or equal to the marginal profit. Similarly, the marginal revenue from selling water must be less than or equal to the marginal profit. The total-profit condition is satisfied if

\[
(4) \quad p^* h_t a_t^* - p(a_t^* - s) - w_t a_t^* - k_i \geq ps
\]

establishing an upper bound of \( \hat{p}_t = p^* h_t - w_t - \frac{w_t}{a_t^*} \). This condition states that the farm’s profit from producing must be greater than or equal to the profit from selling its entire water allocation. In each case, the cutoff is the same for buyers and sellers but varies by technology. For either technology, \( \hat{p}_t < p_i \), and thus the farm’s decision to produce is determined by the total-profit condition. We assume that the probability that \( p \) exceeds \( \hat{p}_t \) is insignificant and therefore limit the analysis to the range of prices for which the farm produces the maximum output with either technology.

We assume that the farm pays nothing for its initial water allocation at time \( t \).\(^2\) The farm’s supply fluctuates stochastically due to changes in weather and public policy. We represent the farm’s stochastic supply process by a geometric Brownian motion, \( ds = \alpha_s dt + \sigma_s dz_s \), where \( s(t) \) is in units of acre-feet per acre.\(^3\) \( \alpha_s \) is the instantaneous drift rate of the supply process, \( \sigma_s \) is the instantaneous variance rate and \( z_s \) is a Wiener process (Dixit and Pindyck).\(^4\)

The farm can smooth its water supply by buying or selling water in the spot market. It can trade a given number of acre-feet for use at time \( t \); however, a spot market in long-term water rights is assumed not to exist. If it did, expected future values of \( s \) might change. The market price of water, in dollars per acre-foot, is represented by a geometric Brownian motion with positive drift, \( dp = \alpha_p p dt + \sigma_p p dz_p \), where \( E[dz_s, dz_p] = \gamma dt \) and \( \gamma < 0 \). Periods of low aggregate water supply correspond to periods of high prices. To the extent that changes in a farm’s supply mirror changes in the aggregate supply, \( s \) and \( p \) will be negatively correlated implying that \( \gamma \) is negative. However, given the existence of a competitive market with no transaction costs, the farm’s investment decision is independent of \( s \).

### The Investment Decision

The farm’s decision to invest in the modern irrigation technology depends on the trade-off between the expected present value of the investment and the fixed cost \( I \) of switching technologies. The value of the investment at a given \( t \) is the increase in the profit flow with the modern technology

\[
(5) \quad v(p) = \pi_i(p, s) - \pi_s(p, s).
\]

Substituting in equation (2) and simplifying, given that \( a_i = a_t^* \) and \( y_i = y^* \), \( i = 1, 2 \), the increase in profit flow is

\[
(6) \quad v(p) = pa^* - q
\]

where \( a^* = a_t^* - a_i^* \) and \( q = w_t a_t^* + k_s - w_t a_i^* - k_i \). \( pa^* \) is the market value of the water conserved and \( q \) is the production cost increase. Since the yield is equivalent with either technology when \( a_t^* \) is applied, both technologies generate the same revenue. Therefore, the increase in profit depends only on the value of the water conserved and the difference in production costs. In addition, while the profit with either technology depends on \( s \), the increase in profit with the modern technology is independent of \( s \).

The farm’s investment decision depends on the expected present value of the increase in profit over all future time periods

\[
(7) \quad V(p) = E \int_0^{\infty} p_t a^* e^{-\rho t} dt - \int_0^{\infty} q e^{-rt} dt.
\]

Because \( p \) is stochastic, it is discounted by the risk-adjusted rate \( \rho \). In contrast, \( q \) is deterministic and is discounted by the risk-free interest rate \( r \). The expected present value can be written as

\[
(8) \quad V(p) = \frac{pa^* - q}{\delta}, \quad \text{where} \quad \delta = \rho - \alpha_p.
\]

In the traditional NPV investment model, the farm should invest if \( V(p) \geq I \), that is,
if the expected present value of the investment is greater than or equal to the fixed cost of investment. The farm trades off the water conservation benefits of the modern technology against the increased operating costs of the modern technology and the required investment cost. In the NPV model, the farm will choose the modern technology if the price of water is greater than or equal to $\tilde{p}$, where

$$\tilde{p} = \frac{\delta}{a^*}(I + \frac{q}{r}).$$

Intuitively, the farm is more likely to choose the modern technology (i.e., $\tilde{p}$ decreases) as the water savings $a^*$ associated with the modern technology increase. The farm is less likely to choose the modern technology at the discount rate $\delta$, the fixed cost of investment $I$, or the additional operating cost $q/r$ increase.

In practice, farms often require that the investment benefits exceed the costs by a positive hurdle rate. The NPV model ignores key aspects of the investment decision that may make farms hesitant to invest. It does not consider uncertainty, irreversibility, and the fact that farms have the option to wait and invest at a later date.

**The Value of the Option to Invest**

If the farm’s water supply falls short, it can buy water in the market instead of investing in modern irrigation technology. The farm has the option to invest if the price of water should rise in the future. Let $F(p)$ represent the value of the farm’s option to invest in the modern technology. Over low price ranges, the expected present value of the investment $V(p)$ is less than the fixed cost of investment $I$. Therefore, the option to switch technologies is “out of the money.” At a sufficiently high water price, however, the option to switch technologies will become “in the money,” and the farm will exercise its option to invest. The farm trades off the benefit of waiting for more information before committing to the investment against the opportunity cost of waiting.

Dynamic optimization techniques are used to derive the investment threshold. Define $\tilde{p}$ to be the price that triggers investment. In the region $(0, \tilde{p})$, in which the farm holds onto its opportunity to invest, the Bellman equation is

$$\rho F(p) dt = E[dF(p)].$$

Using Ito’s Lemma to expand the right-hand side of equation (10), $F(p)$ can be shown to satisfy the following differential equation:

$$\frac{1}{2}\sigma^2 \rho^2 F''(p) + \alpha_p F'(p) - \rho F(p) = 0$$

subject to the boundary conditions

$$(12a) \quad F(0) = 0$$

$$(12b) \quad F(\tilde{p}) = V(\tilde{p}) - I$$

$$(12c) \quad F'(\tilde{p}) = V'(\tilde{p}).$$

Equation (12a) states that when the price of water is zero, the option to invest is worthless. The value-matching condition (12b) states that the value of the option should equal the expected present value less the fixed cost of investment at the threshold. The smooth-pasting condition (12c) states that the change in the value of the option should equal the change in the expected present value of the investment at the threshold.

Solving equation (11) subject to equations (12a–12c), the general solution for the value of the option reduces to

$$F(p) = B_1 p^\beta$$

where $\beta_1$ is the positive root of the fundamental quadratic equation

$$\frac{1}{2}\sigma^2 \rho^2 (p - 1) + (\rho - \delta)\beta - \rho = 0,$$  

and the constant $B_1$ must be determined as part of the solution. Combining equations (8) and (13) with the boundary conditions (12a–12c), the investment threshold is

$$\tilde{p} = \frac{B_1}{\beta_1 - 1} \left( \frac{q}{r} + I \right),$$

where $\beta_1/(\beta_1 - 1)$ is the hurdle rate. Defining $\tilde{I} = q/r + I$ and $\tilde{V}(p) = p\alpha^*/\delta$, the threshold condition can be rewritten as

$$\tilde{V}(\tilde{p}) = \left( \frac{\beta_1}{\beta_1 - 1} \right) \tilde{I}.$$

Because $\beta > 1$, the condition states that the expected revenue from the investment must be greater than the total investment cost at
the threshold. Rearranging equation (14), the threshold price of investment is

\[
\bar{p} = \left( \frac{\beta_1}{\beta_2 - 1} \right) \frac{\delta}{\rho}.
\]

For \( p < \bar{p} \) the farm holds onto its option to invest, and for \( p \geq \bar{p} \) the farm exercises its option and invests in the modern technology. Note that \( \bar{p} = (\beta_1/(\beta_2 - 1))\tilde{p} \), where \( \tilde{p} \) is the threshold price of the NPV model. When one accounts for uncertainty, irreversibility and the option to wait, the farm requires a higher price before it is willing to invest.

**Investment Example**

The following section analyzes the farm’s investment decision for a given set of parameter values. The baseline parameter values, shown in table 1, are intended to be representative of actual values. They are based on estimates obtained by other researchers, and information obtained from California water districts and irrigation consultants (Riddering; Slavin; University of California Committee of Consultants; Westlands Water District). We examine the sensitivity of the hurdle rate, the threshold price and the threshold value of investment to changes in the level of water price uncertainty, the discount rate, the water savings of the modern technology, and the total investment cost.  

**Parameter Values**

Ideally, one would use historical water price data to estimate the water price uncertainty parameter \( \sigma_p \). Unfortunately, good water price data are not available. In most local agricultural water markets, the sale price of water is the private information of the buyer and the seller. Parties to a transaction typically report the transfer to their local water authority, but they are not required to report the price.

**Table 1. Baseline Parameter Values**

<table>
<thead>
<tr>
<th>( \sigma_p )</th>
<th>( \alpha_p )</th>
<th>( \rho )</th>
<th>( I )</th>
<th>( q )</th>
<th>( p_0 )</th>
<th>( \delta )</th>
<th>( \tilde{I} )</th>
</tr>
</thead>
</table>
| 0.15 | 0.06 | 0.05 | $800 \text{ per acre}$ | $20 \text{ per AF}$ | 50 | 0.06 | $I + \frac{\tilde{I}}{\rho} = 1200$

Note: \( \gamma \) is not defined since the investment decision is independent of \( i \) given the assumption of no transaction costs.

The water market in Westlands Water District in California is a prime example. In a typical year, thousands of trades occur within the District, and while the District records the names of each buyer and seller, the date and location of each transfer, the water contract types, and the acre-feet transferred, it does not record the sale prices. The District does record prices, however, when it negotiates purchases from other water districts. Table 2 provides a sample of transfers into Westlands. The price of water varies significantly from year to year depending on the scarcity of water. The price peaked at $223 per AF during the 1991–92 growing season, the last year of a five-year drought (Westlands Water District).\(^6\)

To obtain a value for \( \sigma_p \), it is assumed that with 90% probability the price will remain between $50/3 and $50 x 3 in the next 20 years. Thus, \( \Pr(16.66 \leq p \leq 150) = 0.90 \). Given the geometric Brownian motion price process, changes in \( \ln p \) are normally distributed. Using a log transformation, and the 90% confidence interval for a normal distribution, the variance over the 20-year period is \( \sigma_p^2 \approx (1.10/1.65)^2 = 0.443 \). Dividing by 20, the variance over one year is \( \sigma_p^2 = 0.022 \), which corresponds to a standard deviation, \( \sigma_p \), of 0.15.

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\(^6\) A price of $50 per AF (the initial price in table 1) would require a wet year in Westlands Water District. However, given its junior water rights, the relatively high value of its crops and the barriers to trade that exist in practice, prices in Westlands are higher on average than in most other agricultural regions. The positive drift rate (\( \alpha_p = 0.06 \)) captures the fact that average water prices are increasing over time.
This is the standard deviation used in the base case.

This is likely to be an underestimate of the water price variance in Westlands Water District, but it may be appropriate for other areas. To examine the sensitivity of investment to the degree of water price uncertainty, we examine investment when \( \sigma_p \) equals 0.05 and 0.25. The low value corresponds to the assumption that the price will remain between $35 and $72 per acre-foot with 90% probability over 20 years. This range is conservative. The high value corresponds to the assumption that the price will remain between $8 and $316 per acre-foot with 90% probability. Given the prices in table 2, the high value for \( \sigma_p \) appears to be more realistic for Westlands.

In the baseline case, the risk-adjusted discount rate \( \rho \) is set equal to 12%, and the drift rate on the price of water \( \alpha_p \) is set equal to 6 percent, implying a convenience yield \( \delta = \rho - \alpha_p \) equal to 6%. A higher convenience yield equal to 10% also is evaluated in the sensitivity analysis. We assume a positive drift rate because, while the demand for water has been increasing steadily, average supplies have not increased and have actually decreased in some areas due to stricter environmental regulations that have reallocated water to instream flows.

The water savings \( a^* \) achieved by the modern technology are assumed to be 1.5 AF in the baseline case. We also examine a lower savings level of 1 AF. Caswell, Lichtenberg, and Zilberman estimate that for cotton growers in California’s San Joaquin Valley water use per acre varies between 4.2 and 3.7 AF with furrow irrigation, between 3.1 and 2.8 AF with sprinkler irrigation, and between 2.6 and 2.4 AF with drip irrigation. Thus, the baseline level of 1.5 AF is representative of a switch from furrow to drip technology and the lower level of 1 AF is representative of a switch from furrow to sprinkler technology. The baseline values for the investment cost \( I \) and the increase in operating costs \( q \) are 800 and 20 dollars per acre, respectively. These values were chosen based on interviews with irrigation specialists (Riddering; Slavin). The numbers are consistent with the costs presented by the University of California Committee of Consultants.

We analyze the sensitivity of investment to the level of \( I \) by considering the effect of a 20% investment tax credit.

Baseline Results

In the baseline case, \( V(p) - I \) equals zero when \( p \) equals $48 per AF. This is the threshold price \( \tilde{p} \) associated with the NPV investment rule. In contrast, the threshold price \( \tilde{p} \) derived from the option value rule equals $112 per AF in the baseline case. At this price, the expected net present value of the investment \( V(p) - I \) equals $1,594 per acre. The discrepancy between the two investment rules is large. By failing to account for the influence of uncertainty and irreversibility, the NPV investment rule is biased in favor of early investment.

Other studies have also found large discrepancies between the NPV and option value investment rules. For example, Thurow et al. analyzed the effect of uncertainty and irreversibility on a farm’s incentive to adopt free-stall dairy housing. The housing increases productivity and reduces pollution, but the uncertainty of future environmental regulations deters investment. Using the NPV investment rule, they predicted that a farm would invest in the free-stall technology if the gross expected annual return was greater than or equal to $83,448, but with the option value investment rule, they predicted that a farm would wait until the value was greater than or equal to $190,063.

Sensitivity Analysis and Policy Discussion

The sensitivity results for \( \sigma_p \), \( \delta \), \( a^* \), and \( I \) are summarized in table 3. The first column lists the parameters, and the second column lists the new parameter values. In parentheses it is indicated whether the parameters have increased or decreased relative to the baseline. The third, fourth and fifth columns list the new hurdle rate, threshold price and expected net present value of the investment, respectively.

When \( \sigma_p \) is increased to 0.25, there is a significant increase in the hurdle rate, threshold

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>[\rho_{\tilde{p}} ]</th>
<th>( \bar{\tilde{p}} )</th>
<th>( V(\bar{\tilde{p}}) - I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_p )</td>
<td>0.05 (−)</td>
<td>2.04</td>
<td>98</td>
<td>1,249</td>
</tr>
<tr>
<td>( \sigma_p )</td>
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<td>2.81</td>
<td>135</td>
<td>2,171</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.10 (+)</td>
<td>1.53</td>
<td>122</td>
<td>631</td>
</tr>
<tr>
<td>( a^* )</td>
<td>1.0 (−)</td>
<td>2.33</td>
<td>168</td>
<td>1,594</td>
</tr>
<tr>
<td>( I )</td>
<td>640 (−)</td>
<td>2.33</td>
<td>97</td>
<td>1,382</td>
</tr>
<tr>
<td>Baseline values</td>
<td></td>
<td>2.33</td>
<td>112</td>
<td>1,594</td>
</tr>
</tbody>
</table>
Figure 1. Sensitivity to level of uncertainty

price and expected net present value. At this high level of uncertainty, the farm should wait until \(\hat{V}(\hat{p})\) is 2.8 times greater than \(\hat{I}\) before it invests. The impacts of changes in \(\sigma\), are also illustrated in figure 1.\(^7\) The straight line shows \(V(p) - I\) as a function of \(p\), and the curved line shows \(F(p)\) as a function of \(p\). The points of tangency between \(F(p)\) and \(V(p) - I\) give the threshold price \(\hat{p}\) for each parameter value.

The increase in \(\delta\) reduces the hurdle rate, increases \(\hat{p}\) and reduces \(V(p) - I\). Changes in \(a^*\) and \(I\) do not affect the hurdle rate. However, the reduction in \(a^*\) increases \(\hat{p}\), and the reduction in \(I\) reduces \(\hat{p}\) and \(V(p) - I\) relative to the baseline.

To the extent that increasing modern irrigation technology rates is socially desirable, the sensitivity analysis suggests that policy makers should pursue strategies to reduce water price uncertainty. Some uncertainty is due to stochastic weather conditions, but some is due to inefficient institutions and water conveyance systems. Institutional reforms and improvements in the conveyance networks could reduce uncertainty.

Policies that increase \(a^*\) or reduce \(\delta\) can stimulate investment by inducing farms to adopt the modern irrigation technology at a lower threshold price. In addition to direct policies to reduce \(\delta\), increases in projected water scarcity, reflected in a higher \(\alpha_p\), will reduce \(\delta\) because \(\delta = \rho - \alpha_p\). Finally, reductions in \(\hat{I} = I + q/r\) increase investment incentives. The investment tax credit explored above reduces \(I\), but improvements in technology or management practices that reduce \(q/r\) could also stimulate investment.

Investment Simulations

Figure 2 illustrates one realization of a geometric Brownian motion price process over ten years using the drift and variance rates from table 1 and an initial water price of $50 per AF at \(t = 0\).\(^8\) Figure 2 also shows the price trend, given the same initial price and baseline rate and no uncertainty. As discussed above, in the baseline case the threshold price using the option value investment rule is $112 per AF, while the threshold price using the NPV rule is $48 per AF. Given these threshold prices and the generated price process, the farm would invest under the option value rule after 9 years and 11 months \((t = 119\) months). Using the NPV rule, the farm would invest immediately at \(t = 0\) since the initial price of $50 per AF exceeds the NPV threshold.

Figure 2 demonstrates just one realization of the geometric Brownian motion price process. The price process stays fairly close to

\(^7\) Due to space constraints, the illustrations associated with changes in \(\delta, a^*\) and \(\hat{I}\) have not been included. They are available from the authors on request.

\(^8\) The prices were generated on a monthly basis and therefore, the yearly drift and variance rates were converted to monthly rates. The monthly drift parameter equals 0.005 and the monthly variance parameter equals 0.043.
the trend for the first year and a half before dipping below the trend and then climbing upward. To assess the average behavior of the geometric Brownian motion process, we generated 500 realizations of the price process over a 25-year period using the same initial price, drift rate and variance rate. At the end of ten years, the sample mean price is $92 per AF and the sample standard deviation is $47 per AF. After 25 years, the sample mean price is $228 per AF and the sample standard deviation is $181 per AF. The equivalent trend prices are $91 and $223 per AF after ten and 25 years, respectively. As one would expect, the sample mean price and the trend price begin to diverge as $t$ increases.

The adoption time of 9 years and 11 months illustrated in figure 2 is earlier than the sample average adoption time. 70% of the time the trigger price of $112 per AF is not reached until after 10 years. Fifteen percent of the time the farm still has not invested after 25 years. Given that the farm invests within the 25-year period, the average time to adoption is 12 years and 8 months. The fastest time to adoption is 2 years and 9 months.

The above simulation assumes the water savings $a^*$ associated with the modern technology are 1.5 AF per acre. In practice, the efficiency gains of the modern irrigation technology vary depending on individual farm characteristics such as crop type, soil type and land slope. The following example takes the price process from figure 2 and analyzes the diffusion of the modern technology among farms with different potential efficiency gains. $a^*$ is assumed to be lognormally distributed across farms with a mean of 1.5 AF and a standard deviation of 0.5 AF.

Figure 3 shows the diffusion of the modern irrigation technology and the underlying price process over ten years. After two years, only 13% of the farms have adopted the modern technology. After ten years, 61% have adopted. Farms with efficiency gains greater than or equal to 3.3 AF adopt the modern technology immediately given the initial price of $50 per AF. Farms with efficiency gains less than or equal to 1.45 AF do not adopt the modern technology within the ten-year period. Unlike the smooth diffusion pattern predicted by the NPV model, there are spurts of rapid adoption followed by long periods in which no adoption occurs. From April of the third year to February of the sixth year, water prices increase from $55 to $88 AF, and investment in modern irrigation technology increases by 20%. No adoption occurs from March of the sixth year through May of the eighth year. During this period, the price of water falls as low as $72 per AF.

**Barriers to Trade**

In many areas institutional and technological constraints create high market transaction costs or prevent the formation of water markets all together. If a farm does not have access to a water market, its investment strategy necessarily will be different than that described above. Most importantly, the value of the farm’s investment in modern technology at $t$ will depend on the size of its water
allocation $s(t)$. Without access to a market, a farm with excess water cannot sell it for a profit, and thus has no incentive to adopt modern irrigation technology. On the other hand, a farm with a deficient supply will produce lower yields with the traditional than the modern irrigation technology. Therefore, it may be less likely to wait to invest.

If a farm does not have access to a water market, the value of an investment in modern technology at $t$ is

$$v(s) = \begin{cases} 
-(w, a_t^1 - w, a_t^1) & s \geq a_t^1 \\
-(k_t - k_1) = -q & s < a_t^1
\end{cases}$$

If $s \geq a_t^1$, the farm can produce the maximum output $y^*$ at a lower cost with the traditional than with the modern technology, and it cannot sell its excess supply. Thus $v(s)$ is negative. For all $s < a_t^1$, the farm’s yield is lower with the traditional than the modern technology. As $s$ decreases below $a_t^1$, $v(s)$ increases until it reaches its maximum at $a_t^2$. When $s < a_t^2$, the farm cannot produce the maximum yield with either technology, but its yield is higher with the modern technology than with the traditional technology. We assume that the probability is very small that a farm’s supply will drop below $a_t^2$. Thus, we limit the analysis to $s \geq a_t^2$, the region in which the value of adopting the modern technology increases as $s$ decreases.

The question of interest is how the existence or absence of a market affects a farm’s technology adoption decision. Figure 4 illustrates the timing of investment with and without the existence of a market. Suppose the initial water supply and price are $(s, p)$ as shown in quadrant III. With this quantity-price combination, the farm will use the traditional technology under either the market or the nonmarket system. Under the market scenario the farm will invest in the modern technology if $p$ increases to $\bar{p}$, and under the nonmarket scenario the farm will invest if $s$ falls to $\bar{s}$.

Suppose then the nonmarket scenario is the status quo, and consider the effect of introducing a water market. If $(s, p)$ follows path $A$ into quadrant IV, the market causes the farm to delay adoption. If $(s, p)$ follows path $C$ into quadrant II, the market causes the farm to adopt more quickly. Finally, if $(s, p)$ follows path $B$ into quadrant I, the farm will adopt at the same time with or without a water market. We refer to paths $A$ and $C$ as the “slow” and “fast” paths, respectively.

The relevant path for a given farm may depend on the system of water rights. Aggregate water supplies vary from year to year.
depending on stochastic weather conditions, and these supplies are distributed to farmers according to nonmarket allocation mechanisms. Because price is not used to equate demand with supply when determining initial allocations, the water distribution authority must ration supplies during drought years.

Supplies can be rationed according to a proportional or priority rights system. Under a proportional system, all farms receive a fixed portion of the aggregate supply, and if there is a shortage all share supply reductions equally. Path B might apply in this case. Under a priority system, junior-right holders only receive supplies once the contracts to senior-right holders are met. A junior-right holder would be more likely to experience the slow path, because if aggregate water supplies fall, its supplies might fall significantly while the market price increases only slightly. In contrast, a farm with senior water rights might experience the fast path, since it is insulated against supply cutbacks. Thus, the introduction of a market should induce farms with junior rights (buyers) to adopt more slowly and farms with senior rights (sellers) to adopt more quickly than they would without access to a market.

More generally, whether water markets delay or induce modern technology adoption depends critically on the market price of water. A market will have a greater positive effect on adoption the higher the equilibrium price. As more intersector water markets evolve, in which farms can sell water to urban or environmental users, the market price of water should increase given that the marginal value of water is typically higher in nonagricultural uses. The development of intersectoral water markets should increase the adoption of modern irrigation technology.

**Summary and Conclusions**

This article developed a stochastic dynamic model of irrigation technology adoption. The model predicts that when a farm has access to a water market, it will not invest in modern irrigation technology until the expected present value of investment exceeds the cost of investment by a potentially large hurdle rate.

The article also showed that while water markets may induce adoption by some farms, they may delay adoption by other farms. With a market, a farm has the option to postpone adoption, because it can augment its supplies by purchasing water in the market. All else equal, the introduction of a market is likely to induce farms with abundant supplies to adopt earlier and farms with scarce supplies to adopt later than they would in the absence of a market.

We also analyzed the effects of key parameter changes on a farm’s investment. The size of the investment hurdle rate is especially sensitive to the degree of uncertainty...
in future water prices. The greater the uncertainty, the larger must be the expected benefit before a farm is willing to invest. An increase in the relative efficiency of the modern technology causes the threshold price to decrease, and an increase in the discount rate or the fixed cost of investment causes it to increase. An investment tax credit can increase investment rates by decreasing the fixed cost of investment. However, a relatively large credit is necessary to generate a significant reduction in the threshold price.

The efficiency gains associated with modern irrigation technologies vary across farms depending on crop type, soil type, land slope, and other factors. Using a simulated stochastic price process, the article analyzed the diffusion pattern associated with a distribution of farms. The farms with the greatest efficiency gains were the earliest adopters of modern irrigation technology, and the diffusion pattern was lumpy. There were periods of rapid adoption, in response to water price shocks, followed by years in which no adoption occurred.

The pattern of investment predicted by the model in this article is more consistent with observed investment behavior than the pattern predicted by the NPV model. Given uncertainty, irreversibility and the option to wait, it is rational for farms to wait until the expected benefits of investment exceed the costs by a potentially large hurdle rate before investing in modern irrigation technology.

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